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HIGH VELOCITY DISLOCATIONS

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Research Directorate
AIR FORCE SPECIAL WEAPONS CENTER
Air Force Systems Command
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New Mexico

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(Prepared under Contract No. AF 29(601)-4343
by J. Cotner, A. Van Hull, and J. Weertman,
Department of Materials Science, The Technological Institute, Northwestern University, Evanston, Illinois)

HEADQUARTERS
AIR FORCE SPECIAL WEAPONS CENTER
Air Force Systems Command
Kirtland Air Force Base
New Mexico

ABSTRACT

The elastic displacement field of moving edge dislocations in anisotropic body-centered-cubic and face-centered-cubic crystals is found. From the elastic displacements the shear stress on the dislocation slip plane is determined. The anomalous velocity range in which edge dislocations of like sign attract one another has been calculated for a number of metals and ionic crystals. It is found that anisotropy does not appreciably expand the anomalous range.

The problem of dislocation moving on the interface separating media of different elastic properties has been considered. The anomalous velocity range may or may not exist, depending on the values of the elastic constants in the two media. The dislocation self-energy is infinite at the slowest sound velocity. Supersonic dislocation behavior is qualitatively described.

PUBLICATION REVIEW

This technical documentary report has been reviewed and is approved.

Perry L Hine, Col USAF
for DONALD I. PRICKETT
Colonel USAF
Director, Research Directorate

John J. Dishuck
JOHN J. DISHUCK
Colonel USAF
DCS/Plans & Operations

TABLE OF CONTENTS PAGE

CHAPTER I. INTRODUCTION

This final report describes the analysis and calculations on fast moving dislocations that have been undertaken under this contract. Chief interest in this field has been to study the anomalous velocity region in which edge dislocations of like sign attract rather than repel one another. This velocity region occurs from the Rayleigh wave velocity (approximately 0.9 times the slowest sound velocity) to the velocity at which the dislocation self energy is infinite. In the anomalous velocity region dislocation coalescence can take place. This coalescence could lead to crack formation and fracture.

Chapter III and Chapter IV describe the results found for fast moving dislocations moving in anisotropic body-centered-cubic crystals.

Chapters V and VI present similar work on dislocations moving in face-centered-cubic crystals. Finally, in Chapter VII the results are given of work done on the problem of dislocations moving on the interface separating material of different elastic properties.

All of the work described in this final report has been written for journal articles, and has appeared or will appear in the literature (Weertman, 1962a, 1962b, 1962c; Cotner and Weertman 1962a, 1962b; Van Hull and Weertman, 1962). *

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* References are listed at the end of this report.

More complete details of the analysis and calculations are given in these articles as well as in the Status Reports 1 through 6. **

** Status Reports 1 through 6 are available from AFSWC

CHAPTER II SUMMARY

The main interest in this work was to see if the anomalous velocity region is larger or smaller in anisotropic crystals than it is in isotropic materials. From the results of numerical calculations which were made for a number of metals and ionic crystals it was found that the anomalous region is usually smaller in anisotropic crystals than in isotropic materials. The anomalous region was never found to be appreciably larger than that of isotropic crystals although (Teutonico, 1962a, 1962b, 1962c,) it is known theoretically that it is possible for the region to be very extended. These results are similar to those reached by Teutonico (1962a, 1962b, 1962c) through numerical calculations concerning dislocations different from those considered here.

Since dislocation damping forces are large at ordinary temperatures, it would not appear likely that dislocations would move at velocities lying in the anomalous range. It may be possible to have dislocations moving in the anomalous region at very low temperatures, where dislocation damping forces become small, or at very high stresses such as are encountered in shock loading experiments. If dislocations did move at these velocities, they could contribute to fracture phenomena.

CHAPTER III. Fast Moving Edge Dislocations on the (110) Plane in Anisotropic Body-centered-cubic Crystals.

In this section the problem of an edge dislocation moving uniformly in a b.c.c. lattice is analysed. A dislocation is considered which lies in a (110) plane parallel to a <112> direction and has a Burgers vector in a <111> direction. Edge dislocations of this type are common in b.c.c. crystals.

It has been pointed out (Weertman 1960, 1961) that edge dislocations in an isotropic medium show unusual behavior when they move at high velocity. Above the Rayleigh wave velocity (approximately 0.9 times the transverse sound velocity) the stress on the slip plane of a moving edge dislocation changes sign, and edge dislocations of like sign actually attract rather than repel one another. This anomalous behavior obviously may have importance in fracture phenomena since a coalescence of edge dislocations can lead to crack formation. Chief interest in this section is to investigate the effect that anisotropy has in extending or contracting the velocity range in which the anomalous dislocation behavior occurs.

The problem of a moving edge dislocation was first considered by Eshelby (1949), who found the solution for the elastic displacements in an isotropic medium. Another method of obtaining Eshelby's solution was given later by Radok (1956). Bullough and Bilby (1954) next considered edge dislocations moving in anisotropic crystals. They found the solution for the elastic displacements for the case in which the problem

can be considered as one of plane strain. The dislocation cannot be analyzed as a problem in plane strain, and a more general solution for the elastic displacements must be found. Fortunately, it is quite obvious from Bullough and Bilby's work what the form of the more general solution must be.

THEORY

Since the dislocation under consideration lies parallel or perpendicular to the crystal directions $\langle 111\rangle$, $\langle 110\rangle$ and $\langle 112\rangle$, it is convenient to introduce a coordinate system whose axes run along these directions. Therefore, we shall adopt the right-hand coordinate system in which the positive x axis is the $\langle 110\rangle$ direction, the y axis is the $\langle 112\rangle$ direction, and the z axis is the $\langle 111\rangle$ direction.

When the coordinate axes are chosen parallel to the three $\langle 100\rangle$ directions, a cubic crystal has three independent elastic constants: c_{11}, c_{12} and c_{44}. In a coordinate system rotated from the cube axes the elastic constants are given by other quantities. Hearmon(1956, 1957) has derived explicit formulas for obtaining the elastic constants in any rotated coordinate system. The following constants are obtained from Hearmon's formulas for the coordinate system adopted in the previous paragraph.

$$\begin{bmatrix} c_{11}' & c_{12}' & c_{13}' & c_{14}' & 0 & 0 \\ c_{12}' & c_{11}' & c_{13}' & -c_{14}' & 0 & 0 \\ c_{13}' & c_{13}' & c_{33}' & 0 & 0 & 0 \\ c_{14}' & -c_{14}' & 0 & c_{44}' & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44}' & c_{44}' \\ 0 & 0 & 0 & 0 & c_{14}' & c_{66}' \end{bmatrix}, \quad \ldots (I.1)$$

where

$$\left.\begin{aligned}
c_{11}' &= \tfrac{1}{2}(c_{11} + c_{12} + 2c_{44}), \\
c_{33}' &= \tfrac{1}{3}(c_{11} + 2c_{12} + 4c_{44}), \\
c_{44}' &= \tfrac{1}{3}(c_{11} - c_{12} + c_{44}),; \\
c_{66}' &= \tfrac{1}{6}(c_{11} - c_{12} + 4c_{44}) = \tfrac{1}{2}(c_{11}' - c_{12}'), \\
c_{12}' &= \tfrac{1}{6}(c_{11} + 5c_{12} - 2c_{44}), \\
c_{13}' &= \tfrac{1}{3}(c_{11} + 2c_{12} - 2c_{44}), \\
c_{14}' &= \frac{1}{3\sqrt{2}}(-c_{11} + c_{12} + 2c_{44}) = \sqrt{2}(c_{33}' - c_{11}') \\
c_{14}' &= \sqrt{2}(c_{66}' - c_{44}') = \sqrt{2}(c_{12}' - c_{13}').
\end{aligned}\right\} \quad (I.2)$$

It is the appearance of the constant c_{14}' in the matrix (I.1) which makes it impossible to apply Bullough and Bilby's solution to the dislocation under study.

The equations of dynamic equilibrium, which must be satisfied in any problem in elasticity, are

$$\frac{\rho \partial^2 u_i}{\partial t^2} = \frac{\partial}{\partial x_j}\left\{c'_{ijkl}\,\frac{1}{2}\left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right)\right\} \quad \text{(summed on } j,k,l,)\ldots\text{(I.3)}$$

where ρ is the density of the crystal; t is time; x_i stands for x,y or z; u_i stands for the elastic displacements u, v and w in the x,y and z directions respectively; and c'_{ijkl} are the elastic constants in the uncontracted notation.

Since the dislocation being considered lies parallel to the y axis, the derivatives with respect to y can be set equal to zero; otherwise the elastic displacements would be a function

of distance along the dislocation line. Another simplification in the equilibrium equations can be made for a dislocation moving at a uniform velocity V in the z direction. The solution of any of the three elastic displacements u_i will be of the form $u_i = u_i\,(x, z-Vt)$. Hence in eqns. (I.3) the time derivative $\partial^2/\partial t^2$ can be replaced by $V^2\,(\partial^2/\partial z^2)$. Equations (I.3) are now reduced to the following set:

$$\rho V^2 \frac{\partial^2 u}{\partial z^2} = c_{11}' \frac{\partial^2 u}{\partial x^2} + c_{44}' \frac{\partial^2 u}{\partial z^2} + 2c_{14}' \frac{\partial^2 v}{\partial x \partial z} + (c_{13}' + c_{44}') \frac{\partial^2 w}{\partial x \partial z} \quad \text{(I.4a)}$$

$$\rho V^2 \frac{\partial^2 v}{\partial z^2} = c_{66}' \frac{\partial^2 v}{\partial x^2} + c_{44}' \frac{\partial^2 v}{\partial z^2} + c_{14}' \frac{\partial^2 w}{\partial x^2} + 2c_{14}' \frac{\partial^2 u}{\partial x \partial z}, \quad \text{(I.4b)}$$

$$\rho V^2 \frac{\partial^2 w}{\partial z^2} = c_{33}' \frac{\partial^2 w}{\partial z^2} + c_{44}' \frac{\partial^2 w}{\partial z^2} + c_{14}' \frac{\partial^2 v}{\partial x^2} + (c_{13}' + c_{44}') \frac{\partial^2 u}{\partial x \partial z} \quad \text{(I.4c)}$$

In contrast to the case of an edge dislocation moving in an isotropic medium or to the problem considered by Bullough and Bilby, it is impossible to eliminate one of these equations. All three elastic displacements are involved in the displacement solution.

It is obvious from the results of Bullough and Bilby that, for the dislocation being considered, the solution of elastic displacements which will satisfy eqns. (I.4) very probably has the form:

$$u = \frac{b}{4\pi}\left[A_1 \log(z'^2 + \gamma_1^2 x^2) + A_2 \log(z'^2 + \gamma_2^2 x^2) + A_3 \log(z'^2 + \gamma_3^2 x^2)\right] \quad \text{(I.5a)}$$

$$v = \frac{b}{2\pi}\left[B_1\tan^{-1}\frac{\gamma_1 x}{z'} + B_2\tan^{-1}\frac{\gamma_2 x}{z'} + B_3\tan^{-1}\frac{\gamma_3 x}{z'}\right] \ldots\ldots \text{(I.5b)}$$

$$w = \frac{b}{2\pi}\left[C_1\tan^{-1}\frac{\gamma_1 x}{z'} + C_2\tan^{-1}\frac{\gamma_2 x}{z'} + C_3\tan^{-1}\frac{\gamma_3 x}{z'}\right] \ldots\ldots \text{(I.5c)}$$

In this equation $z' = z - Vt$, b is the length of the Burgers vector, and A_1, B_1, C_1, γ_1, etc. are constants.

To evaluate eqns. (I.5) which contain 12 unknown constants, it is necessary to find 12 independent equations in these constants. Nine such equations are obtained by setting eqns. (I.5) into eqns. (I.4). For example, if eqns. (I.5) are placed in eqn. (I.4a), the following equation results:

$$\sum_{i=1}^{3} \frac{(z'^2-\gamma_i^2x^2)}{(z'^2\ \gamma_i^2x^2)^2}\left[(c_{44}' - c_{11}'\gamma_i^2 - \rho V^2)A_i + 2c_{14}'\gamma_i B_i + \gamma_i(c_{13}'+c_{44}')C_i\right] = 0 \quad \text{(I.6)}$$

Since this equation must hold for all values of z' and x, the bracket term containing A_i, B_i, etc., must equal zero for each value of i. Thus from eqn. (1.6) one obtains three equations in the unknown constants. Similar substitutions into eqns. (I.4b) and (I.4c) produce six more equations. The nine equations so obtained are:

$$(c_{44}' - c_{11}'\gamma_i^2 - \rho V^2)A_i + 2c_{14}'\gamma_i B_i + \gamma_i(c_{13}' + c_{44}')C_i = 0 \quad \text{(I.7a)}$$

$$\gamma_i(c_{13}' + c_{44}')A_i + \gamma_i^2 c_{14}' B_i - (c_{33}' - \rho V^2 - c_{44}'\gamma_i^2)C_i = 0 \quad \text{(I.7b)}$$

$$2\gamma_i c_{14}' - (c_{44}' - c_{66}'\gamma_i^2 - \rho V^2)B_i + \gamma_i^2 c_{14}' C_i = 0 \quad \text{(I.7c)}$$

(where i = 1, 2 or 3).

Since the right-hand side of each of eqns. (I.7) is zero, the determinant made up of the coefficients of A_i, B_i, etc. must equal zero to permit all the equations to be valid simultaneously. In other words, γ_i^2 must be one of the three roots of the cubic equation

$$(\gamma_i^2 - \bar{\gamma}_1^2)(\gamma_i^2 - \bar{\gamma}_2^2)(\gamma_i^2 - \bar{\gamma}_3^2) = \left(\frac{c_{14}'^2\gamma_i^2}{c_{44}'c_{66}'c_{11}'}\right)\left[4\gamma_i^2 c_{13}' - \gamma_i^2 c_{44}' + 4c_{33}' - \rho V^2(4 - \gamma_i^2) + c_{11}'\gamma_i^4\right] \qquad \text{(I.8)}$$

where

$$\bar{\gamma}_1^2 = \frac{c_{44}' - \rho V^2}{c_{66}'}$$

and

$$\bar{\gamma}_2^2, \bar{\gamma}_3^2 = \left(\frac{1}{2c_{44}'c_{11}'}\right)\Big\{c_{11}'(c_{33}' - \rho V^2) + c_{44}'(c_{44}' - \rho V^2) - (c_{44}' + c_{13}')^2 \pm \Big[\{c_{11}'(c_{33}' - \rho V^2) + c_{44}'(c_{44}' - \rho V^2) - (c_{44}'+c_{13}')^2\}^2 - 4c_{44}'c_{11}'(c_{33}' - \rho V^2)(c_{44}' - \rho V^2)\Big]^{1/2}\Big\}$$

($\bar{\gamma}_2^2$ corresponds to the minus sign and $\bar{\gamma}_3^2$ to the plus sign.) Equation (I.8) determines 3 of the 12 unknown constants.

EVALUATION OF THE CONSTANTS A_i, B_i, and C_i.

The constants A_i, B_i, C_i can be evaluated if all three roots γ_1^2 are positive real numbers or if one of these roots is a positive real number and the other two are complex conjugates. Either of these two possibilities will be met in the velocity range from $V = 0$ to the velocity at which the dislocation self-energy becomes infinite.

When two of the roots, say γ_1^2 and γ_2^2, are complex conjugates (and hence γ_1 and γ_2 are likewise complex conjugates) it is only necessary that the pairs of constants A_1 and A_2, B_1 and B_2 and C_1 and C_2, also be complex conjugate numbers to ensure that the elastic displacements given by eqn. (1.5) are real numbers.

To evaluate the nine other unknown constants appearing in eqns.(I.5),(I.6) and (I.7), it is necessary to obtain three additional independent equations containing these constants. So far the condition that the displacements must describe a dislocation has not been used. That is, if a circuit is made around the dislocation line, the net displacement must equal the Burgers vector. In eqns. (I.5) the log terms return to their original values after a complete circuit is made around the dislocation line. The arc tan terms, however, change by a factor 2π. The Burgers vector of the dislocation being considered lies parallel to the z direction. Hence it can be seen from eqns. (I.5) that the following two equations must be satisfied

$$C_1 + C_2 + C_3 = 1 \quad . \quad . \quad . \quad . \quad (I.9)$$

and

$$B_1 + B_2 + B_3 = 0 \quad . \quad . \quad . \quad . \quad (I.10)$$

Only one more equation is needed. It is obtained from the condition that point forces must not exist at the core of the dislocation. To eliminate the point forces it is first necessary to determine the stresses produced by the elastic displacements of eqns. (I.5). The following set of equations gives the stresses

σ_{ij} in the coordinate system being used if all derivatives with respect to y are zero:

$$\left.\begin{aligned}
\sigma_{xx} &= c_{11}'\frac{\partial u}{\partial x} + c_{14}'\frac{\partial v}{\partial z} + c_{13}'\frac{\partial w}{\partial z}, \\
\sigma_{xy} &= c_{14}'\frac{\partial u}{\partial z} + c_{66}'\frac{\partial v}{\partial x} + c_{14}'\frac{\partial w}{\partial x}, \\
\sigma_{xz} &= c_{44}'\frac{\partial u}{\partial z} + c_{14}'\frac{\partial v}{\partial x} + c_{44}'\frac{\partial w}{\partial x}, \\
\sigma_{yz} &= c_{14}'\frac{\partial u}{\partial x} + c_{44}'\frac{\partial v}{\partial z}, \\
\sigma_{zz} &= c_{13}'\frac{\partial u}{\partial x} + c_{33}'\frac{\partial w}{\partial z}, \\
\sigma_{yy} &= c_{12}'\frac{\partial u}{\partial x} - c_{14}'\frac{\partial v}{\partial z} + c_{13}'\frac{\partial w}{\partial z}.
\end{aligned}\right] \quad . \; . \; . \; . \; (I.11)$$

To ensure that no point force acts at the core of the dislocation in a direction perpendicular to the slip plane, it is only necessary that

$$\int_{-\infty}^{\infty} \sigma_{xx}\,dz = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \; (I.12)$$

If this integral were not equal to zero a point force would have to be applied to the core of the dislocations. The magnitude of this force would be equal to the integral. When eqns. (I.5) are substituted intc eqn. (I.11), one obtains the following expression for σ_{xx}

$$\sigma_{xx} = \frac{b}{2\pi}\sum_{i=1}^{3}\left(\frac{\gamma_i x}{z'^2 \quad \gamma_i^2 x^2}\right)\left[c_{11}'\gamma_i A_i - c_{14}'B_i - c_{13}'C_i\right] \quad (I.13)$$

Equation (I.14) is the result of substituting this expression into eqn. (I.12) (and also of using eqns. (I.9) and (I.10)):

$$\gamma_1 A_1 + \gamma_2 A_2 + \gamma_3 A_3 = \frac{c_{13}'}{c_{11}'} \quad . \; . \; . \; . \; . \; (I.14)$$

We now have a sufficient number of equations to determine A_i, B_i and C_i. Equations (I.7), (I.9), (I.10) and (I.14) can be rearranged into the following three sets of equations, from which it is possible to determine all the constants A_i, B_i and C_i. The summation in these equations is from i = 1 to i = 3.

$$\sum B_i = 0, \quad \ldots \quad \text{(I.15a)}$$

$$\sum B_i/\gamma_i^2 = \frac{c_{14}'(c_{44}' + \rho V^2)}{(c_{44}' - \rho V^2)^2}, \quad \ldots \quad \text{(I.15b)}$$

$$\sum B_i\gamma_i^2 = \frac{-c_{14}}{c_{44}'c_{66}' - c_{14}'^2}\left[c_{33}' - \rho V^2 \frac{c_{13}'}{c_{11}'}(c_{44}' - c_{13}')\right]. \quad \text{(I.15c)}$$

$$\sum C_i = 1, \quad \ldots \quad \text{(I.16a)}$$

$$\sum C_i/\gamma_i^2 = \frac{-c_{44}'(c_{13}' + \rho V^2)}{(c_{33}' - \rho V^2)(c_{44}' - \rho V^2}, \quad \ldots \quad \text{(I.16b)}$$

$$\sum C_i\gamma_i^2 = \frac{-2c_{13}'}{c_{11}'} + \frac{c_{66}'}{c_{44}'c_{66}' - c_{14}'^2}\left[c_{33}' - \rho V^2 + \frac{c_{13}'}{c_{11}'}(c_{44}' - c_{13}')\right] \quad \text{(I.16c)}$$

$$\sum A_i\gamma_i = \frac{c_{13}'}{c_{11}'} \quad \ldots \quad \text{(I.17a)}$$

$$\sum A_i/\gamma_i = \frac{-c_{44}'}{c_{44}' - \rho V^2} \quad \ldots \quad \text{(I.17b)}$$

$$\sum A_i\gamma_i^3 = \frac{c_{13}'}{c_{11}'^2}(c_{44}' - \rho V^2) + \frac{2c_{14}'}{c_{11}'}\sum B_i\gamma_i^2 + \frac{(c_{13}' + c_{44}')}{c_{11}'}\sum C_i\gamma_i^2 \quad \text{(I.17c)}$$

The solutions of eqns. (I.15), (I.16) and (I.17) are:

$$B_1 = \alpha(\gamma_2^2 - \gamma_3^2)(b_3\gamma_1^2 + b_2\gamma_1^2\gamma_2^2\gamma_3^2), \quad \text{..........(I.18a)}$$

$$B_2 = \alpha(\gamma_3^2 - \gamma_1^2)(b_3\gamma_2^2 + b_2\gamma_1^2\gamma_2^2\gamma_3^2), \quad \text{..........(I.18b)}$$

$$B_3 = \alpha(\gamma_1^2 - \gamma_2^2)(b_3\gamma_3^2 + b_2\gamma_1^2\gamma_2^2\gamma_3^2), \quad \text{..........(I.18c)}$$

where

$$\alpha^{-1} = (\gamma_1^2 - \gamma_2^2)(\gamma_3^2 - \gamma_2^2)(\gamma_3^2 - \gamma_1^2),$$

$$b_2 = \sum B_i/\gamma_i^2,$$

$$b_3 = \sum B_i\gamma_i^2,$$

$$C_1 = \alpha(\gamma_2^2 - \gamma_3^2)(g_3\gamma_1^2 + g_2\gamma_1^2\gamma_2^2\gamma_3^2 - \gamma_1^2\gamma_2^2 - \gamma_1^2\gamma_3^2), \quad \text{(I.19a)}$$

$$C_2 = \alpha(\gamma_3^2 - \gamma_1^2)(g_3\gamma_2^2 + g_2\gamma_1^2\gamma_2^2\gamma_3^2 - \gamma_2^2\gamma_1^2 - \gamma_2^2\gamma_3^2), \quad \text{(I.19b)}$$

$$C_3 = \alpha(\gamma_1^2 - \gamma_2^2)(g_3\gamma_3^2 + g_2\gamma_1^2\gamma_2^2\gamma_3^2 - \gamma_3^2\gamma_1^2 - \gamma_3^2\gamma_2^2), \quad \text{(I.19c)}$$

where

$$g_2 = \sum C_i/\gamma_i^2$$

$$g_3 = \sum C_i\gamma_i^2,$$

$$A_1 = \alpha\gamma_1^{-1}(\gamma_2^2 - \gamma_3^2)(a_3\gamma_1^2 + a_2\gamma_1^2\gamma_2^2\gamma_3^2 - a_1\gamma_1^2(\gamma_2^2 + \gamma_3^2)), \quad \text{(I.20a)}$$

$$A_2 = \alpha\gamma_2^{-1}(\gamma_3^2 - \gamma_1^2)(a_3\gamma_2^2 + a_2\gamma_1^2\gamma_2^2\gamma_2^2 - a_1\gamma_2^2(\gamma_1^2 + \gamma_3^2)), \quad \text{(I.20b)}$$

$$A_3 = \alpha\gamma_3^{-1}(\gamma_1^2 - \gamma_2^2)(a_3\gamma_3^2 + a_2\gamma_1^2\gamma_2^2\gamma_3^2 - a_1\gamma_3^2(\gamma_1^2 + \gamma_2^2)), \quad \text{(I.20c)}$$

where

$$a_1 = c_{13}' / c_{11}',$$

$$a_2 = \sum A_i / \gamma_i,$$

$$a_3 = \sum A_i\gamma_i^3.$$

A consideration of these equations will show that it is indeed true that when γ_1 and γ_2 are complex conjugates, the constants A_1 and A_2, etc., also are complex conjugates. Equations (I.18), (I.19) and (I.20), along with eqn. (I.8), give the general solution of our problem in the velocity range from V = 0 to the velocity at which the dislocation self-energy becomes infinite.

SHEAR STRESS ON THE DISLOCATION SLIP PLANE

Of all the stresses around an edge dislocation, the stress which is of primary interest is that stress which produces a force on another edge dislocation with a parallel Burgers vector. This force is $\sigma_{xz}b$, where b is the Burgers vector. Hence σ_{xz} is the shear stress of greatest interest to us. This stress can be found by substituting eqns. (I.5) into eqn. (I.11). One obtains

$$\sigma_{xz} = \frac{b}{2\pi}\sum_{i=1}^{3} \frac{z'}{z'^2 + \gamma_i^2 x^2} \left(c_{44}'A_i + \gamma_i c_{14}'B_i + \gamma_i c_{44}'C_i\right) \quad \text{(I.21a)}$$

The shear stress on the slip plane of the dislocations itself (x=0) is

$$(\sigma_{xz})_{x=0} = \frac{b}{2\pi z'} \sum \left(c_{44}' A_i + \gamma_i c_{14}' B_i + \gamma_i c_{44}' C_i\right) \quad \text{(I.21b)}$$

In the case of a stationary or slowly moving dislocation this stress can be expected to be positive for a positive edge dislocation. Slowly moving dislocations of like sign and on the same slip plane repel one another. It is known (Weertman 1961) that an edge dislocation moving in an isotropic medium experiences a shear stress on its slip plane which decreases with increasing dislocation velocity, and which actually can change sign. In the velocity region where the

sign of the shear stress has been reversed, dislocations of like sign attract rather than repel one another. The velocity separating this region of abnormal behavior from the normal region is that velocity at which the shear stress is exactly equal to zero. This velocity turns out to be the Rayleigh wave velocity.

To ensure the existence of an abnormal region in which dislocations of like sign attract one another it is necessary only that eqn.(I.21b) goes to zero at a velocity lower than the velocity at which one of the roots γ_i^2 is equal to zero or at which two of the roots become negative. The velocity at which eqn. (I.21b) is equal to zero is the velocity for which

$$\sum_i (c_{44}'A_i + \gamma_i c_{14}'B_i + \gamma_i c_{44}'C_i) = 0 \quad . \quad . \quad . \quad . \quad (I.22a)$$

The velocity satisfying this equation is the velocity at which a generalized Rayleigh wave would travel on a (110) plane in a <111> direction. Since the shear stress σ_{xy}, in general, is not equal to zero in the dislocation slip plane, an actual surface wave cannot propagate on a (110) plane in a <111> direction in an anisotropic crystal.

Equation (I.21a) can be simplified somewhat through the use of eqn. (I.7b). If the latter equation is divided through by γ_i and then summed from $i = 1$ to $i = 3$ and the result substituted into eqn. (I.21a), one obtains

$$\sum_i \left[c_{13}'A_i - (c_{33}' - \rho V^2)\, C_i \gamma_i^{-1}\right] = 0 \quad .. \quad .. \quad . \quad (I.22b)$$

This equation has the advantage that all the B_i constants have been eliminated. If eqns. (I.19) and(I.20) are substituted into eqn. (I.22b) the following equation results:

$$f_1(\gamma_1^2 + \gamma_2^2 + \gamma_3^2 + \gamma_1\gamma_2 + \gamma_2\gamma_3 + \gamma_3\gamma_1)$$

$$+ f_2\gamma_1\gamma_2\gamma_3(\gamma_1 + \gamma_2 + \gamma_3) - f_3 = 0 \quad \ldots \quad \ldots \quad \ldots \quad (\text{I.22c})$$

where

$$f_1 = c_{13}'a_1 - (c_{33}' - \rho V^2) = \frac{c_{13}'^2}{c_{11}'} - (c_{33}' - \rho V^2),$$

$$f_2 = c_{13}'a_2 - (c_{33}' - \rho V^2)g_2 = \frac{c_{44}'\rho V^2}{c_{44}' - \rho V^2},$$

$$f_3 = c_{13}'a_3 - (c_{33}' - \rho V^2)g_3 = -\frac{c_{13}'^2}{c_{11}'^2}(c_{44}' + 2c_{13}' + \rho V^2),$$

$$+ 2(c_{33}' - \rho V^2)\frac{c_{13}'}{c_{11}'} + \left[\frac{(c_{33}' - \rho V^2) + (c_{13}' / c_{11}')(c_{44}' - c_{13}')}{c_{44}'c_{66}' - c_{14}'^2}\right]$$

$$\left[\frac{c_{13}'}{c_{11}'}(c_{66}'c_{13}' + c_{66}'c_{44}' - 2c_{14}'^2) - c_{66}'(c_{33}' - \rho V^2)\right].$$

The principal goal of this section was the drivation of eqn. (I.22c). With this equation, one can investigate the effect of anisotropy on the extent of the velocity region in which anomalous dislocation behaviour occurs. For an anomalous velocity region to exist it is necessary, of course, that the velocity which satisfies eqn. (I.22) is smaller than the velocity at which the roots γ_1^2 first become zero or negative. The dislocation energy is infinite at this latter velocity and hence it is the limiting velocity of dislocation motion in normal circumstances. In Chapter IV results of numerical calculations using Eqn. (I.22) are presented.

CHAPTER IV. Calculations for Body-Centered-Cubic Crystals

This section presents the results of a numerical calculation of the shear stress on the slip plane of an edge dislocation moving uniformly in a body-centered cubic lattice. The dislocation considered is moving on a (110) plane in a <111> direction. The primary concern is to investigate the extent of the anomalous velocity region in which the shear stress reverses sign and dislocations of like sign attract one another. This anomalous behavior of the dislocations, which leads to a coalescence of fast moving dislocations, may be expected to be of importance in fracture phenomena. Iron, therefore, is logically the metal to study.

In the previous section, formulas were developed which permit the calculation of the shear stress on the slip plane of an edge dislocation moving on a (110) plane. It was found that the shear stress σ acting on the slip plane at a distance z' from the center of a dislocation moving with the velocity V is

$$\sigma = -\frac{b}{2\pi z'} \Big[f_1(\gamma_1^2 + \gamma_2^2 + \gamma_3^2 + \gamma_1\gamma_2 + \gamma_2\gamma_3 + \gamma_3\gamma_1) + f_2\gamma_1\gamma_2\gamma_3(\gamma_1 + \gamma_2 + \gamma_3) - f_3 \div (\gamma_1+\gamma_2)(\gamma_2+\gamma_3)(\gamma_3+\gamma_1) \Big] \qquad \text{(II.1)}$$

where b is the Burgers vector and

$$f_1 = \frac{c_{13}'^2}{c_{11}'} - (c_{33}' - \rho V^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \text{(II.2a)}$$

$$f_2 = c_{44}'\rho V^2/(c_{44}' - \rho V^2) \quad .. \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(II.2b)}$$

$$f_3 = \frac{c_{13}'^2}{c_{11}'^2}(c_{44}' + 2c_{13}' + \rho V^2) + \frac{2c_{13}'}{c_{11}'}(c_{33}' - \rho V^2)$$

$$+ \left[\frac{(c_{33}' - \rho V^2) + \frac{c_{13}'}{c_{11}'}(c_{44}' - c_{13}'}{c_{44}'c_{66}' - c_{14}'^2}\right]$$

$$\times \left[\frac{c_{13}'}{c_{11}'}\ (c_{66}'c_{13}' + c_{66}'c_{44}' - 2c_{14}'^2)\right] - c_{66}'(c_{33}' - \rho V^2) \quad \text{(II.2c)}$$

and $\gamma_1^{\ 2}$, $\gamma_2^{\ 2}$ and $\gamma_3^{\ 2}$ are the three roots of the cubic equation given by Eqn. (I.8).

RESULTS FOR IRON

In Table I.1 are listed values of these elastic constants for iron calculated from data of Seitz and Read given in Hearmon's review article (1946).

These values were used to calculate these roots of equation (I.3), which are tabulated in Table II.2.

The limiting dislocation velocity is found to be 0.941c, where $c = \sqrt{(c_{44}'/\rho)}$ = the velocity of shear waves in a <111> direction in a cubic crystal.

The limiting velocity of dislocation motion in iron is smaller than the shear wave velocity in the direction of dislocation motion. The possibility that the dislocation motion may be limited to a velocity smaller than the shear wave velocity first was pointed out by L. J. Teutonico (1962a, 1962b, 1962c).

The shear stress on the slip plane, plotted in Fig.II.1 was calculated from equation (I.1) by using the values of the

TABLE I.1

Elastic constants of iron (in units of 10^{11}dyn/in^2.)

c_{11}	c_{12}	c_{14}
23.7	14.1	11.6

c_{11}'	c_{33}'	c_{12}'	c_{13}'	c_{44}'	c_{66}'	c_{14}'
30.5	32.8	11.8	9.56	7.06	9.33	3.40

TABLE II.2

$\frac{\rho V^2}{c_{44}'}$	γ_3^2	γ_1^2 and γ_2^2
0	4.77	0.359 ± 0.278i
0.1	4.61	0.325 ± 0.268i
0.2	4.46	0.277 ± 0.250i
0.3	4.33	0.226 ± 0.231i
0.4	4.18	0.180 ± 0.213i
0.5	4.04	0.133 ± 0.191i
0.6	3.90	0.086 ± 0.169i
0.7	3.76	0.038 ± 0.135i
0.8	3.62	-0.008 ± 0.0925i
0.82	3.59	-0.020 ± 0.0797i
0.84	3.57	-0.029 ± 0.0710i
0.86	3.54	-0.038 ± 0.0520i
0.88	3.53	-0.048 ± 0.03229i

TABLE II.3

$\frac{\rho V^2}{c_{44}'}$	$\frac{\sigma z'}{c_{44}' b}$
0	0.269
0.1	0.259
0.2	0.251
0.3	0.241
0.4	0.229
0.5	0.214
0.6	0.190
0.7	0.160
0.8	0.0814
0.82	0.0568
0.84	0.00925
0.86	-0.0795
0.88	-0.315

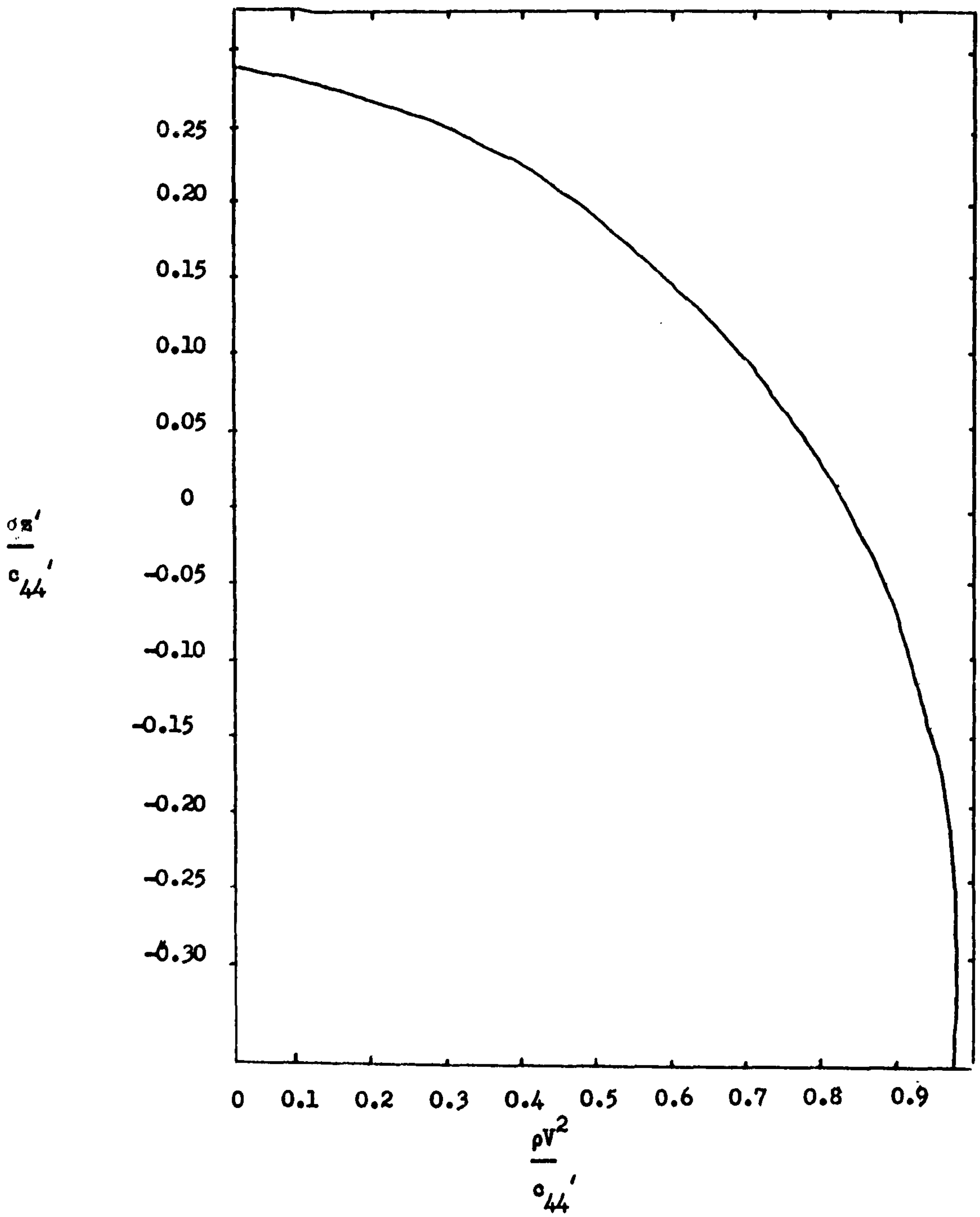

FIG. II.1. Plot of shear stress σ on slip plane versus $\rho V^2/c_{44}'$

TABLE II.4

Calculated Characteristic Dislocation Velocities. *

Material	c_i	c_r	Δc	$\dfrac{c_{14}'}{c_{44}'}$
		Body-centered-cubic crystals		
Li	0.809	0.803	0.006	1.042
β Brass	0.824	0.815	0.009	1.010
Na	0.869	0.855	0.014	0.965
K	0.853	0.842	0.011	0.905
αFe	0.941	0.918	0.023	0.485
W	1.000	0.913	0.087	0
Mo	1.000	0.903	0.097	-0.115
CsI	1.000	0.835	0.165	-0.156
CsBr	1.000	0.895	0.105	-0.167
RbBr	0.754	0.710	0.044	-0.439
RbI	0.726	0.693	0.033	-0.464
αFe[a]	0.952	0.927	0.026	0.439
αFe[b]	0.949	0.923	0.024	0.433
Li[c]	0.809	0.804	0.005	1.042

*Velocities are expressed in units of $(c_{44}'/\rho)^{1/2}$ = transverse sound velocity in the<111> direction. Elastic data used in calculations are those listed in Huntington's review article (1958) for Li, K, Na (data of Bender), W, Mo, and β brasss (data of Artman and Thompson); in Reintz's paper (1961) for CsI, CsBr, RbBr , and RbI; and in Hearmon's review article (1946) for αFe (data of Seitz and Read).

(a) Calculated by L,J.Teutonico (private communication) using elastic data listed in Hearmon (1946) (data of Kimura & Ohno)

(b) Calculated by L.J.Teutonico (private communication)using elastic data of Rayne and Chandrasekhar (1961) (footnote cont. on next page)

roots of equation (I.8). The actual calculated values of the shear stress are listed in Table II.3. It can be seen that the stress is positive at low velocities, goes through zero and then becomes negative at high velocities. The velocity at which the stress is zero is the Rayleigh wave velocity. Its value is 0.918c.

The result that the anomalous velocity region extends from only 0.918c to 0.941c is in qualitative agreement with calculations (Weertman, 1962a) made for dislocations moving in slightly anisotropic lattices. From these calculations it was found that a slight anisotropy (with c_{14}' taken to be a positive number) decreases the extent of the anomalous velocity region. An isotropic crystal with the same c_{44}'/c_{33}' ratio as iron has a Rayleigh wave velocity of the order of 0.94c. The limiting velocity of dislocation motion for an isotropic crystal is always c. Hence the extent of the anomalous velocity range for edge dislocations moving on the (110) plane in iron is 3 times smaller than that of an isotropic crystal with comparable values of c_{44}' and c_{33}'. Thus it is more difficult in iron to bring dislocations moving on a (110) plane into a velocity range where dislocation coalescence can take place.

RESULTS ON OTHER BODY-CENTERED-CUBIC CRYSTALS

Table II.4 indicates the principal results on calculation on other b.c.c. crystals. This table lists the velocity c_1 at which tne self-energy of a moving dislocation is infinite, the velocity c_r

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(footnote continued from previous page)
(c) Calculated by L.J. Teutonico with elastic data listed in Huntington (1958).

(the generalized Rayleigh wave velocity) at which the shear stress on the dislocation slip plane is zero, the difference between these velocities $\Delta c = c_i - c_r$, and the ratio of the elastic constant c_{14}'/c_{44}' where c_{14}' and c_{44}' are elastic constants in the rotated coordinate system. The ratio c_{14}'/c_{44}' is a measure of the degree of anisotropy of the elastic constants. The quantity c_{14}' is equal to zero for an isotropic material. Listed in Table II.4 are the results of calculations by Teutonico on alpha iron and lithium.

In the velocity range from c_r to c_i dislocations on the same slip plane of like sign attract rather than repel one another. In this velocity range dislocation behavior is anomalous.

From calculations of the effect of a slight anisotropy (Weertman, 1962a) on the extent of the anomalous velocity range it was predicted that when c_{14}' is positive, an increase in the anisotropy decreases the anomalous velocity range, whereas if c_{14}' is negative, the anomalous range increases. An inspection of Table 11.4 reveals that crystals with a positive c_{14}' do have a smaller Δc than tungsten, which is an almost isotropic material. The crystals CsI and CsBr, which have a negative c_{14}', have a larger Δc than tungsten. However, RbBr and RbI, which are more anisotropic than CsI and CsBr and which also have a negative c_{14}', have a smaller Δc than tungsten. It is clear that the results of the slightly anisotropic calculations cannot be extrapolated to large values of anisotropy and that there is a limit to the extent to which anisotropy can widen the anomalous velocity range.

CHAPTER V. Fast Moving Edge Dislocation on the (111) Plane in Anisotropic Face-Centered-Cubic Crystals

This section considers the problem of edge dislocations moving uniformly in anisotropic face-centered-cubic crystals. Considered is the ordinary edge dislocation of face-centered-cubic crystals which lie in a (111) plane parallel to a $\langle 112 \rangle$ direction and have a Burgers vector in a $\langle 110 \rangle$ direction. In the following section the elastic displacement field of this type of dislocation will be determined and also the shear stress on the dislocation slip plane. The anomalous velocity range can then be determined from this shear stress.

ELASTIC DISPLACEMENT FIELD

The coordinate system is adopted in which the x axis is parallel to the $\langle 110 \rangle$ direction, the y axis is parallel to the $\langle \bar{1}12 \rangle$ direction, and the z axis is parallel to the $\langle 1\bar{1}1 \rangle$ direction. The elastic constants in this coordinate system are the same as those given in Chapter III.

Since the dislocation being considered lies parallel to the y axis, the elastic displacements about the dislocation must be independent of y. Since a dislocation moving uniformly with a velocity V in the x direction is also being considered, the elastic displacements will be a function of x - Vt, where t is the time. If u, v, and w represent the elastic displacements in the x, y, and z directions respectively, the equations of dynamic equilibrium reduce to the following:

$$\rho V^2 \frac{\partial^2 u}{\partial x^2} = c_{11}' \frac{\partial^2 u}{\partial x^2} + c_{44}' \frac{\partial^2 u}{\partial z^2} + 2c_{14}' \frac{\partial^2 v}{\partial x \partial z} + (c_{13}' + c_{44}') \frac{\partial^2 w}{\partial x \partial z} \qquad \text{(III.1a)}$$

$$\rho V^2 \frac{\partial^2 v}{\partial x^2} = c_{66}' \frac{\partial^2 v}{\partial x^2} + c_{44}' \frac{\partial^2 v}{\partial z^2} + c_{14}' \frac{\partial^2 w}{\partial x^2} + 2c_{14}' \frac{\partial^2 u}{\partial x \partial z} \qquad \text{(III.1b)}$$

$$\rho V^2 \frac{\partial^2 w}{\partial x^2} = c_{33}' \frac{\partial^2 w}{\partial z^2} + c_{44}' \frac{\partial^2 w}{\partial x^2} + c_{14}' \frac{\partial^2 v}{\partial x^2} + (c_{13}' + c_{44}') \frac{\partial^2 u}{\partial x \partial z} \qquad \text{(III.1c)}$$

It is obvious from the results of Bullough and Bilby (1954) that the solution of the elastic displacements which satisfies Eqns. (III.1) is very likely to have the form:

$$u = \frac{b}{2\pi}\left(A_1 \tan^{-1}\frac{\gamma_1 z}{x'} + A_2 \tan^{-1}\frac{\gamma_2 z}{x'} + A_3 \tan^{-1}\frac{\gamma_3 z}{x'}\right) \qquad \text{(III.2a)}$$

$$v = \left[\frac{b}{4\pi} \; B_1 \log(x'^2 + \gamma_1^2 z^2) + B_2 \log(x'^2 + \gamma_2^2 z^2) + B_3 \log(x'^2 + \gamma_3^2 z^2)\right] \qquad \text{(III.2b)}$$

$$w = \frac{b}{4\pi}\left[C_1 \log(x'^2 + \gamma_1^2 z^2) + C_2 \log(x'^2 + \gamma_2^2 z^2) + C_3 \log(x'^2 + \gamma_3^2 z^2)\right] \qquad \text{(III.2c)}$$

where $x' = x - Vt$, b is the length of the Burgers vector, and A_1, B_1, γ_1, etc., are constants. These equations contain 12 unknown constants which must be determined. Equations (III.2) are similar to a set which give the elastic displacements about an edge dislocation moving on a (110) plane in a body-centered-cubic crystal. The 12 unknown constants can be evaluated in a manner similar to that

carried out in the body-centered-cubic case. This evaluation has been carried out by Weertman (1962b).

SHEAR STRESS ON THE DISLOCATION SLIP PLANE

The stress which produces a force on another parallel edge dislocation with a parallel Burgers vector is the shear stress σ_{xz}.

$$\sigma_{xz} = c_{44}'\frac{\partial u}{\partial z} + c_{14}'\frac{\partial v}{\partial z} + c_{44}'\frac{\partial w}{\partial x} \tag{III.3}$$

which with appropriate substitutions, becomes

$$\sigma_{xz} = \frac{b}{2}\sum_{1}^{3}\frac{x'}{x'^2 + \gamma_i^2 z^2}(c_{44}'\gamma_i A_i + c_{14}'B_i + c_{44}'C_i) \tag{III.4}$$

On the slip plane of the dislocation itself ($z = 0$) this equation reduces to

$$\sigma_{xz} = \frac{b}{2\pi x'}\sum_{1}^{3}(c_{44}'\gamma_i A_i + c_{14}'B_i + c_{14}'C_i) \tag{III.5a}$$

which also can be written as

$$\sigma_{xz} = \frac{b}{2\pi x'}\sum_{1}^{3}\left[(c_{11}' - \rho V^2)\gamma_i^{-1}A_i \;-\; c_{14}'B_i \qquad c_{13}'C_i\right] \tag{III.5b}$$

If the values of A_i, B_i and C_i (Weertman 1962b) are substituted in Eq.(III.5b) one finds:

$$\sigma_{xz} = \frac{b}{2\pi x'}\,\frac{h_1(\gamma_1^2+\gamma_2^2+\gamma_3^2+\gamma_1\gamma_2+\gamma_2\gamma_3+\gamma_3\gamma_1)+h_2+h_3\gamma_1\gamma_2\gamma_3(\gamma_1+\gamma_2+\gamma_3)}{(\gamma_1+\gamma_2)(\gamma_2+\gamma_3)(\gamma_3+\gamma_1)} \tag{III.5c}$$

where

$$h_1 = c_{11}' - \rho V^2 - \frac{c_{14}'^2}{c_{44}'} - \frac{c_{13}'^2}{c_{33}'}$$

$$h_2 = -(c_{11}' - \rho V^2)a_2 + c_{14}'b_2 + c_{13}'g_2$$

$$h_3 = \frac{-\rho V^2 \left[c_{44}'(c_{66}' - \rho V^2) - c_{14}'^2\right]}{(c_{44}' - \rho V^2)(c_{66}' - \rho V^2) - c_{14}'^2}$$

At the Rayleigh wave velocity the shear stress σ_{xz} equals zero. At velocities below the Rayleigh wave velocity σ_{xz} will be positive and above it this stress will be negative. In the velocity range where σ_{xz} is negative, dislocations of like sign attract rather than repel one another. Thus, the velocity at which σ_{xz} is zero is of great interest; the behavior of dislocations moving slower than this velocity is normal whereas dislocations moving faster than this velocity exhibit abnormal behavior.

LIMITING VELOCITY OF DISLOCATION MOTION

As the velocity of a dislocation increases, its self-energy also increases. At some velocity the self-energy will become infinite, and this velocity, therefore, sets an upper limit to dislocation motion. It was previously noted (Weertman 1962a) that the limiting dislocation velocity is the smallest velocity at which one of the roots, γ_1^2, first becomes zero (if all roots are real numbers) or becomes a real but negative number (if two of the roots are complex conjugates). The velocities at which the roots become zero can be found by setting γ_1^2 equatl to zero. Thus one can obtain the following equation:

$$(c_{66}' - \rho V^2)(c_{44}' - \rho V^2)(c_{11}' - \rho V^2) = c_{14}'^2(c_{11}' - \rho V^2). \quad \text{(III.6)}$$

The smallest velocity satisfying this equation is the velocity at

which one of the roots γ_1^2 first becomes zero. The smallest velocity satisfying Eqn.(III.6) is given by the quadratic equation

$$(\rho V^2)^2 - \rho V^2(c_{66}' + c_{44}') + c_{44}'c_{66}' - c_{14}'^2 = 0 \qquad \text{(III.7a)}$$

whose solution is

$$\rho V^2 = \frac{1}{2}(c_{44}' + c_{66}') - \frac{3}{2\sqrt{2}} |c_{14}'| \qquad \text{(III.7b)}$$

If c_{14}' is positive ($c_{66}' > c_{44}'$) this equation can be written as

$$\rho V^2 = c_{44}' - c_{14}' - c_{14}' \Big/ \sqrt{2} = c_{66}' - \sqrt{2}c_{14}' \qquad \text{(III.7c)}$$

If c_{14}' is negative ($c_{44}' > c_{66}'$)

$$\rho V^2 = c_{66}' + c_{14}' \Big/ \sqrt{2} = c_{44}' + \sqrt{2}\, c_{14}' \qquad \text{(III.7d)}$$

The velocities s_1 and s_2, of the two shear waves in <110> direction are (Waterman 1959)

$$s_1 = (c_{44}/\rho)^{1/2} \qquad \text{(III.8a)}$$

$$s_2 = \left[(c_{11} - c_{12})/\rho\right]^{1/2} \qquad \text{(III.8b)}$$

In terms of c_{44}', c_{66}', and c_{14}' these equations become

$$s_1 = \left[(c_{44}' + \sqrt{2}\, c_{14}')/\rho\right]^{1/2} = \left[(c_{66}' + c_{14}'/\sqrt{2})/\rho\right]^{1/2} \qquad \text{(III.8c)}$$

$$s_2 = \left[(c_{44}' - c_{14}'/\sqrt{2})/\rho\right]^{1/2} = \left[(c_{66}' - \sqrt{2}\, c_{14}')/\rho\right]^{1/2} \qquad \text{(III.8d)}$$

If c_{14}' is positive, s_2 is the slower shear wave velocity, and if c_{14}' is negative, s_1 is the slower. From Eqns. (III.7c) and (III.7d), it can be seen that a root γ_1^2 first becomes zero at the slower of the two shear wave velocities in the <110> direction. In

the slightly anisotropic case the roots γ_i^2 are always positive numbers when the dislocation velocity is less than either s_1 or s_2 but is not too close to zero. Hence, for this case the slower of the two shear wave velocities is the limiting velocity of dislocation motion. In the general anisotropic case, two of the roots can be complex conjugates and the dislocation limiting velocity will be the smallest velocity at which either complex conjugate roots turn into a negative number, or a root becomes equal to zero.

CHAPTER VI. Calculations for Face-Centered-Cubic Crystals

In Chapter V an equation was derived which gave the shear stress acting on the slip plane of a moving edge dislocation in an anisotropic face-centered-cubic lattice. This section presents the results of numerical calculations of the shear stress on the slip plane using that equation. The metals which have been investigated are aluminum, copper, gold, silver, lead, and nickel. These calculations are complementary to those on body-centered-cubic crystals.(Chapter IV)

Table IV.1, lists the elastic constant data used in the calculations. These data are taken from Huntington's review article.(1958).

In Table IV.2 are listed calculated values of the shear stress on the slip plane at various velocities. These shear stresses are plotted as a function of dislocation velocity in Figs. IV.1 and IV.2. The most striking result contained in these figures and the table is the extreme narrowness of the velocity range in which the shear stress is negative. (It is in this range that dislocations of like sign attract one another, and thus dislocation coalescence can take place). Aluminum, however, is an exception; here there is an appreciable velocity region in which the shear stress is negative. Aluminum has such an extended anomalous region simply because it is an almost isotropic material.* It is known (Weertman, 1961) that

*Huntington lists another set of elastic constants for aluminum which are slightly more anisotropic than those used in the present paper. We carried out calculations using this more anisotropic data and concluded that the anomalous region is not appreciably reduced in size. It was found that $c = 0.944$ and $v_r = 0.919$(in velocity units of $(c_{44}'/\rho)^{1/2}$.

TABLE IV.1

Elastic constants of metals[a] (In units of 10^{11} dynes/cm^2)

	Al	Cu	Au	Pb	Ni	Ag
c_{11}	10.8	16.8	18.6	4.66	24.7	12.4
c_{12}	6.13	12.1	15.7	3.97	14.7	9.34
c_{44}	2.85	7.54	4.20	1.44	12.5	4.61
c_{11}'	11.3	22.0	21.4	5.73	32.2	15.5
c_{33}'	11.5	23.8	22.3	6.09	34.7	16.5
c_{44}'	2.51	4.08	2.37	0.73	7.46	2.56
c_{66}'	2.68	5.81	3.28	1.08	9.97	3.58
c_{12}'	5.96	10.4	14.8	3.56	12.2	8.31
c_{13}'	5.79	8.68	13.9	3.21	9.72	7.29
c_{14}'	0.24	2.45	1.30	0.50	3.54	1.45

a Data taken from Huntington (1958)

TABLE IV.2

Calculated values of the shear stress on the slip plane at various velocities.

Nickel

$\frac{\rho V^2}{c_{44}'}$	$\frac{\sigma x'}{b c_{44}'}$
0	0.263
0.1	0.255
0.2	0.246
0.3	0.236
0.4	0.223
0.5	0.204
0.6	0.161
0.64	0.0883
0.65	0.0368
0.66	−0.123
0.665	$-\infty$

TABLE IV.2 (Continued)

Lead

$\frac{\rho V^2}{c_{44}}$,	$\frac{\sigma x'}{b c_{44}}$,
0	0.324
0.1	0.317
0.2	0.310
0.3	0.301
0.4	0.286
0.5	0.186
0.505	0.119
0.507	0.0418
0.5075	-0.00348
0.508	-0.0721
0.50928	$-\infty$

TABLE IV.2 (Continued)

Gold

$\frac{\rho V^2}{c_{44}'}$	$\frac{\sigma x'}{bc_{44}'}$
0	0.330
0.1	0.325
0.2	0.315
0.3	0.305
0.4	0.301
0.5	0.271
0.6	0.147
0.608	0.0225
0.61275	$-\infty$

TABLE IV.2 (Continued)

Aluminum

$\frac{\rho V^2}{c_{44}'}$	$\frac{\sigma x'}{b c_{44}'}$
0	0.255
0.1	0.247
0.2	0.237
0.3	0.226
0.4	0.213
0.5	0.197
0.6	0.176
0.7	0.144
0.8	0.099
0.87	-0.0103
0.9	-0.156
0.93303	$-\infty$

TABLE IV.2 (Continued)

Copper

$\frac{\rho V^2}{c_{44}'}$	$\frac{\sigma x'}{b c_{44}'}$
0	0.313
0.1	0.312
0.2	0.306
0.3	0.300
0.4	0.291
0.5	0.261
0.56	0.161
0.57	0.093
0.572	-0.08370
0.574	-0.119
0.57591	$-\infty$

TABLE IV.2 (Continued)

Silver

$\frac{\rho V^2}{c_{44}}$,	$\frac{\sigma x'}{bc_{44}}$
0	0.316
0.1	0.310
0.2	0.304
0.3	0.296
0.4	0.286
0.5	0.268
0.58	0.212
0.59	0.133
0.595	0.0134
0.59839	$-\infty$

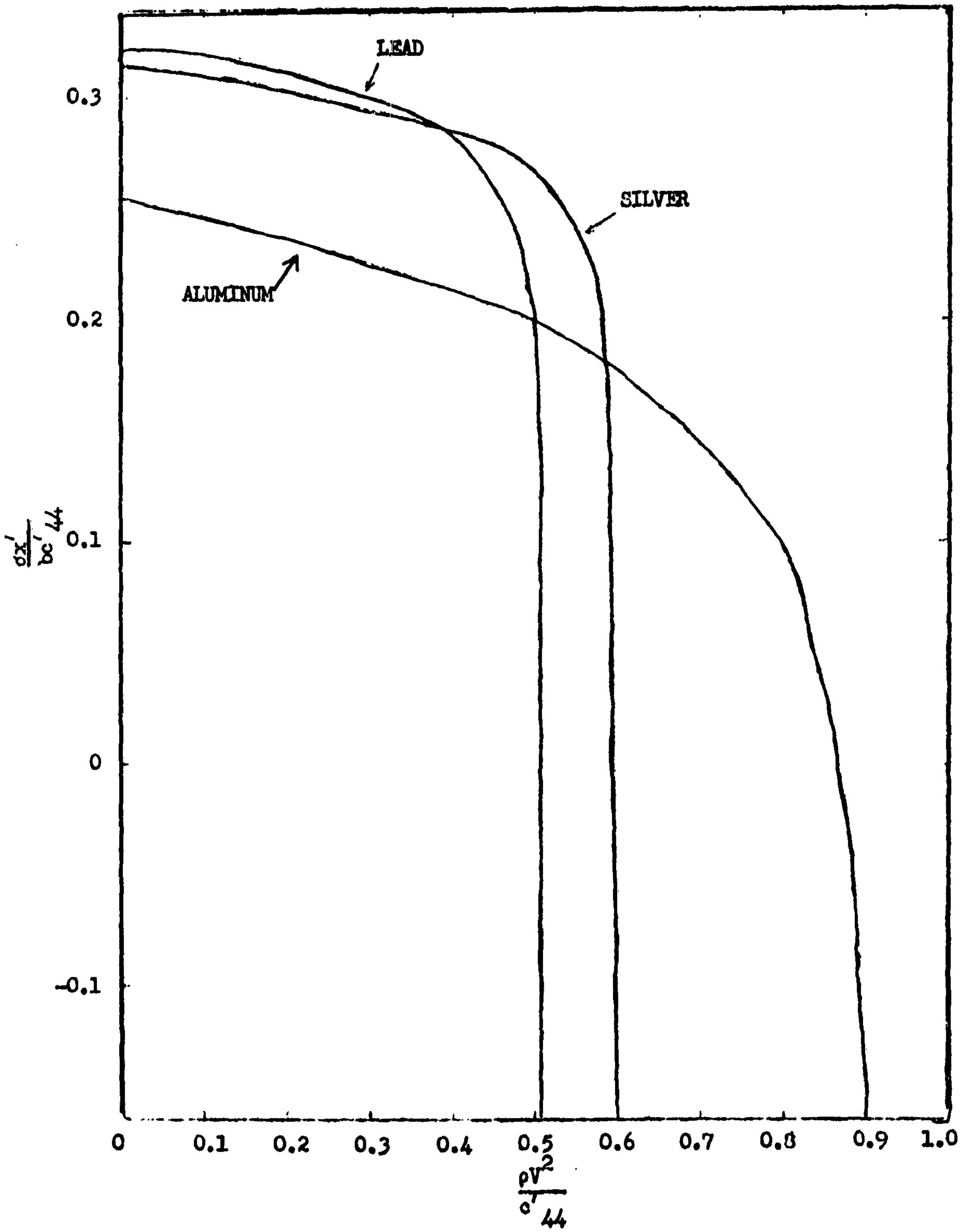

FIG. IV.2. Plot of $\sigma x'/bc_{44}'$ vs $\rho V^2/c_{44}'$ for aluminum, silver, and lead.

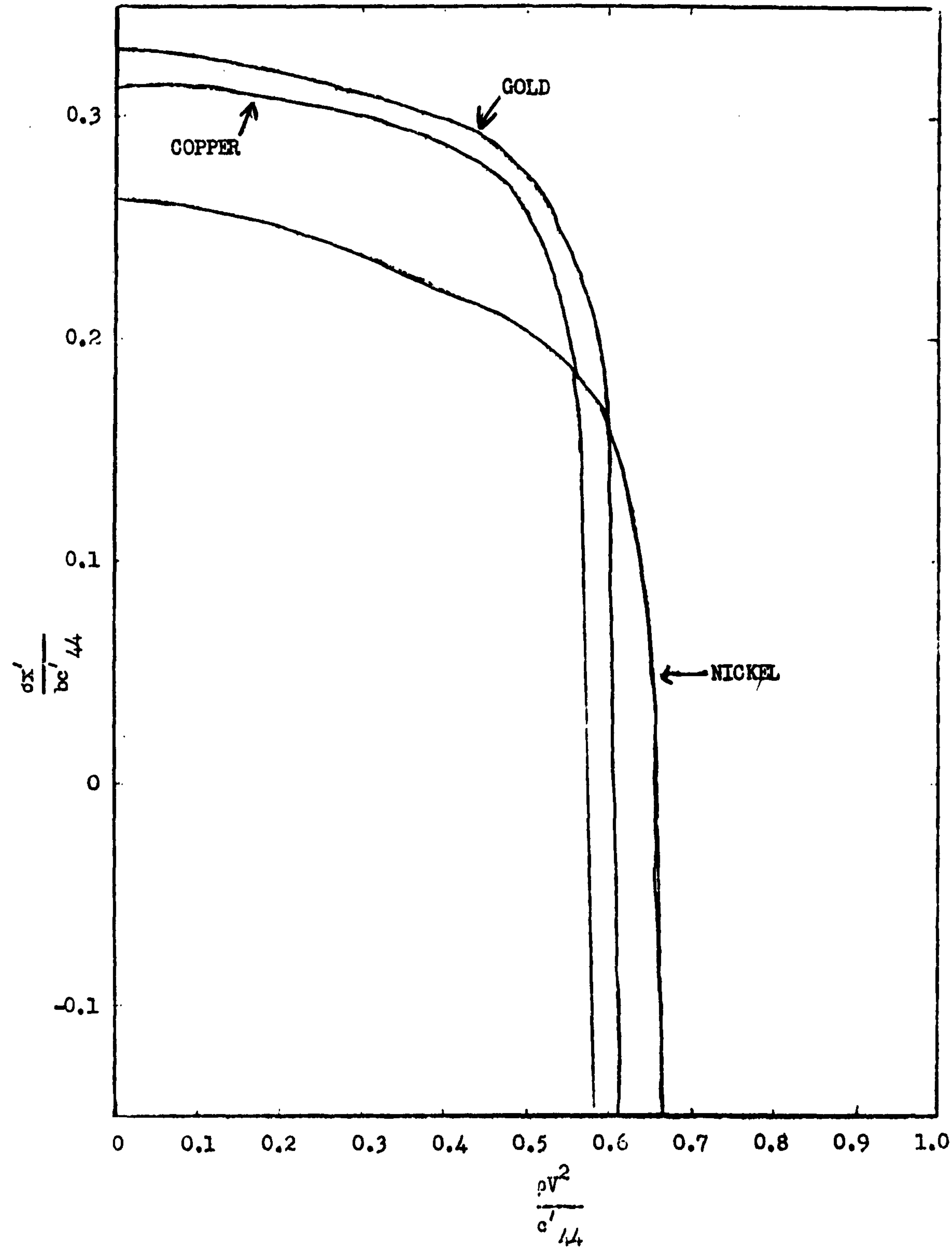

FIG: IV.1 Plot of $\sigma x'/bc_{44}'$ vs $\rho V^2/c_{44}'$ for copper, gold, and nickel.

an extensive anomalous velocity region exists in isotropic materials.

A slight anisotropy usually decreases (Weertman, 1962a, 1962b) the anomalous velocity region. Therefore, it is not surprising that the anomalous region is decreased in extent by a large anisotropy. A reduction was found in the case of alpha iron (Cotner & Weertman, 1962a, 1962b) which is a strongly anisotropic body-centered-cubic metal. However, the degree of decrease in the strongly anisotropic fcc metals is much larger than it is in alpha iron. To illustrate this point, Table IV.3 lists the velocity at which the anomalous region starts (v_r = the Rayleigh wave velocity) and ends (c = the velocity at which the dislocation energy is infinite).

From Table IV.3 it is clear that to bring an edge dislocation into the velocity region in which dislocation coalescence occurs would be more difficult in the case of strongly anisotropic fcc metals than of strongly anisotropic bcc metals. It would be tempting to ascribe the greater ease of fracture of bcc metals to this difference. However, the fact that aluminum has a large anomalous region and yet appears to be ductile down to low temperatures, presents a serious difficulty to such a theory.

TABLE IV.3[a]

Metal	c	v_r
αFe	0.94	0.92
Al	0.966	0.933
Cu	0.759	0.756
Au	0.783	0.780
Pb	0.714	0.712
Ni	0.815	0.806
Ag	0.774	0.771

a Velocities are expressed in units of $(c_{44}'/\rho)^{1/2}$

CHAPTER VII: Dislocations Moving Uniformly on the Interface Between Two Isotropic Media of Different Elastic Properties.

The problem of a dislocation moving on the interface separating two media of different elastic properties is interesting both from the theoretical as well as the practical viewpoint. Diffusionless transformations in crystals probably involve dislocations running on the interfaces between transformed and untransformed material. Since the amount of energy released in such transformations may be large, high dislocation velocities are to be expected. In fact, Eshelby (1956) has proposed that dislocations may run at supersonic velocities in diffusionless transformations.

A dislocation running on a transformation interface is moving on an interface which separates two materials of differing elastic properties and densities. It seems worthwhile to obtain the solution of the stress field about such moving dislocations. This section will attempt to solve the problem for the simplest case: that in which the two elastic media are isotropic. (The assumption of isotropy precludes a treatment of the twinning dislocation. In isotropic materials a twinning dislocation is merely an ordinary dislocation. However, the analysis may have some qualitative application to twinning dislocations in anisotropic material.) Because the supersonic velocity range may be of practical importance the dislocation behavior in this velocity region will be considered qualitatively. Consideration is based on the elucidating analyses

of supersonic dislocations by Eshelby (1956) and Stroh (1962).

THEORY

The solution of the problem being considered can be obtained by extending the known solutions of dislocations moving in isotropic material (Eshelby, 1949; Frank, 1949; Leibfried and Dietz, 1949). Consider a coordinate system in which a dislocation line lies parallel to the x axis and moves on its slip plane in the x direction. Let μ_1, λ_1, and ρ_1 represent the Lamé constants and the density of the material above the slip plane ($y > 0$), and μ_2, λ_2, and ρ_2 the same constants for materials below the slip plane ($y < 0$). In a moving screw dislocation the following equations of dynamic equilibrium have to be satisfied:

$$\frac{\partial^2 w_i}{\partial x^2} + \frac{\partial^2 w_i}{\partial y^2} = \frac{1}{c_i^2} \frac{\partial^2 w_i}{\partial t^2} \tag{V.1}$$

where $i = 1$ or 2, w_1 and w_2 are the elastic displacements in the z direction above and below the slip plane, $c_i = (\mu_i/\rho_i)^{1/2}$ = the transverse sound velocity in each of the two media, and t stands for time.

A moving edge dislocation involves the following equations of dynamic equilibrium:

$$(\lambda_i + 2\mu_i) \frac{\partial^2 u_i}{\partial x^2} + \mu_i \frac{\partial^2 u_i}{\partial y^2} + (\lambda_i + \mu_i) \frac{\partial^2 v_i}{\partial x \partial y} = \rho_i \frac{\partial^2 u_i}{\partial t^2} \tag{V.2}$$

$$(\lambda_i + 2\mu_i) \frac{\partial^2 v_i}{\partial y^2} + \mu_i \frac{\partial^2 v_i}{\partial x^2} + (\lambda_i + \mu_i) \frac{\partial^2 u_i}{\partial x \partial y} = \rho_i \frac{\partial^2 v_i}{\partial t^2}$$

where u_i and v_i are the elastic displacements in the x and y directions respectively. The time differential $\frac{\partial^2}{\partial t^2}$ can be replaced by $V^2 \frac{\partial^2}{\partial x^2}$ for the dislocations moving with a uniform velocity V in the x direction.

From the results of analyses of dislocations moving in isotropic material we anticipate that the solution of Eqns. (V.1) and (V.2) for moving dislocations will be:

(a) Screw Dislocation

$$w_i = \frac{b}{2\pi}\left[G_i \tan^{-1} \frac{\beta_i y}{x'}\right] + E_i \qquad \text{(V.3)}$$

(b) Edge Dislocation

$$u_i = \frac{b}{2\pi}\left[A_i \tan^{-1} \frac{\gamma_i y}{x'} + B_i \tan^{-1} \frac{\beta_i y}{x'}\right] + F_i \qquad \text{(V.4a)}$$

$$v_i = \frac{b}{4\pi}\left[C_i \log (x'^2 + \gamma_i^2 y^2) + D_i \log(x'^2 + \beta_i^2 y^2)\right] + H_i \qquad \text{(V.4b)}$$

In these equations b is the length of the Burgers vector, A_i, B_i, etc., are constants, $\beta_i = (1-V^2/c_i^2)^{1/2}$, $\gamma_i = (1-V^2/c_{\lambda i}^2)$ where $c_{\lambda i} = \left[(\lambda_i+2\mu_i)/\rho_i\right]^{1/2}$ = the longitudinal sound velocity, and $x' = x-Vt$ where V is the velocity of the dislocation. (The constant H_i is added merely to make Eqn. (V.4b) dimensionally correct. The constants E_i and F_i are added in order to match suitably the elastic displacements w_i and u_i across the interface $y = 0$. Since only the differentials of the displacements are important the

constants E_i, F_i and H_i can be ignored).

One should note that at velocities such that β_i (or γ_i) is an imaginary number the arc tan of $\beta_i y/x$ (or $\gamma_i y/x$) can also be written as $\frac{1}{2}\sqrt{-1}\,\log|(x'+|\beta_i|y)/(x'-|\beta_i|y)|$ + a constant. The evaluation of the constants for the screw and for the edge dislocations are considered separately in the following sections.

SCREW DISLOCATIONS

The constants G_i of equation (V.3) are simple to evaluate. If a complete circuit is made around a dislocation the elastic displacement must change by an amount equal to the Burgers vector. Thus from the properties of the arc tan function it is evident that $G_1 + G_2 = 2$ when the dislocation velocity V is less than either c_1 or c_2. Another equation in G_1 and G_2 can be obtained from the condition that the value of the stress must be continuous across the slip plane. Thus at z = 0 the stress σ_{yz} must satisfy the condition $(\sigma_{yz})_1 = (\sigma_{yz})_2$. Now

$$(\sigma_{yz})_i = \mu_i\left(\frac{\partial w_i}{\partial y}\right) = \frac{\mu_i b}{2\pi}\;\frac{G_i\beta_i x'}{x'^2 + \beta_i^2 y^2} \tag{V.6}$$

Therefore

$$\mu_1 G_1 \beta_1 = \mu_2 G_2 \beta_2 \tag{V.7}$$

The only other stress which exists around the screw, namely σ_{xz}, is given by:

$$(\sigma_{xz})_i = \mu_i\left(\frac{\partial w_i}{\partial x}\right) = -\frac{\mu_i b}{2\pi}\;\frac{G_i\beta_i y}{x'^2 + \beta_i^2 y^2} \tag{V.8}$$

and is equal to zero on the slip plane regardless of the values of G_1 and G_2.

A solution of equations (V.5) and (V.7) results in:

$$G_1 = \frac{2\mu_2\beta_2}{\mu_1\beta_1 + \mu_2\beta_2} \qquad \text{(V.9a)}$$

and

$$G_2 = \frac{2\mu_1\beta_1}{\mu_1\beta_1 + \mu_2\beta_2} . \qquad \text{(V.9b)}$$

Equations (V.9) and (V.3) give the solution of the elastic displacement field about the moving screw dislocation. From this field both the stresses (eqns. (V.6) and (V.8)) and the displacement velocities can be found. Once these quantities are known the strain energy and the kinetic energy can be calculated in the usual manner (Eshelby, 1949; Frank, 1949; Leibfried and Dietz, 1949; Weertman, 1961). This calculation gives for the total self-energy E of a screw dislocation moving on the interface between two media each of width R in the y direction and extending to $\pm\infty$ in the x direction

$$E = \left(\frac{b^2}{2}\log\frac{R}{b}\right)\frac{\mu_1\mu_2}{(\mu_1\beta_1 + \mu_2\beta_2)^2}\left\{\frac{\mu_2{\beta_2}^2}{\beta_1} + \frac{\mu_1{\beta_1}^2}{\beta_2}\right\} \qquad \text{(V.10)}$$

As would be expected this expression becomes infinite at the slowest of the two transverse sound velocities (i.e. when β_1 or β_2 is equal to zero.)

SUPERSONIC SCREW DISLOCATION

Two-dimensional supersonic dislocation were treated first by Eshelby (1956)*. Stroh (1962) later more generally treated supersonic dislocations moving in anisotropic media. The main difficulty encountered in analyzing supersonic dislocations is the occurrence, when linear elasticity theory is used, of infinities in the equations. This problem occurs, for example, in eqns. (V.6) and (V.8) at a velocity so large that β_1 is an imaginary number. According to these equations the stresses are infinite along the planes $x'^2 = \beta_1{}^2 y^2$. Stroh (as well as Thomson (1961)) points out that these infinities actually do not occur. Because of the discrete atomic nature of a crystal lattice, only finite stresses and finite energies can exist. It could be assumed that linear elasticity theory holds only for those stresses whose "effective shear stress" τ (defined, for example, by Nye (1957) as the square root of $\frac{1}{2}\sum'_{ij} (\sigma'_{ij})^2$ where $\sigma'_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}$, δ_{ij} is the Kroneker delta, and σ_{ij} is the usual stress component) does not exceed the value τ_o. Screw dislocations when only the stresses σ_{yz} and σ_{xz} are present may be expressed

$$\sigma_{y}{}^2{}_z + \sigma_{x}{}^2{}_z + \tau_o{}^2 \qquad \text{(V.11)}$$

The breakdown in linear elasticity theory in supersonic situations occurs along the planes $x' = \pm \mid \beta_1 \mid y$ when β_1 becomes imaginary. These discontinuities in the displacements and stresses are the plane waves which Eshelby and Stroh have shown must exist in order to maintain a supersonic dislocation.

Figure V.1 shows schematically the behavior of the Eshelby-Stroh

*One-dimensional supersonic dislocations were studied earlier by Frank and van der Merwe (1950)

discontinuity planes as the velocity of a screw dislocation is increased. The lines in this figure represent the region where the stress equals the maximum stress τ_o. For velocities less than c_1 the stress is equal to τ_o only in the core region. At the velocity c_1 the Eshelby-Stroh discontinuity appears. The normal to this discontinuity, which moves with the velocity c_1, is in the direction of the dislocation and hence the discontinuity dissipates no energy to the crystal surface. At a somewhat greater velocity two E-S discontinuities appear. The directions of their normals are different from the direction of dislocation motion and hence surface tractions are required to maintain the discontinuities. If suitable surface tractions are not applied, the dislocation still can run but the slip plane must be able to give up energy. It could do so either if an external shear stress is applied to the crystal or if the slip plane is also a transformation plane which gives up energy as the dislocation runs along it. When no surface tractions are applied the E-S discontinuities which are ahead of the discontinuity behind the dislocations will occur. This E-S discontinuity sends energy out to the crystal surface and an equal amount of energy must be supplied at the dislocation core if the dislocation is to continue to run.

It is simple to obtain an estimate of this energy dissipation. The E-S discontinuity, which is expected to leave a width of the order of the atomic spacing, has associated with it an energy per unit area of the order of $\tau_o b$. The discontinuity makes an

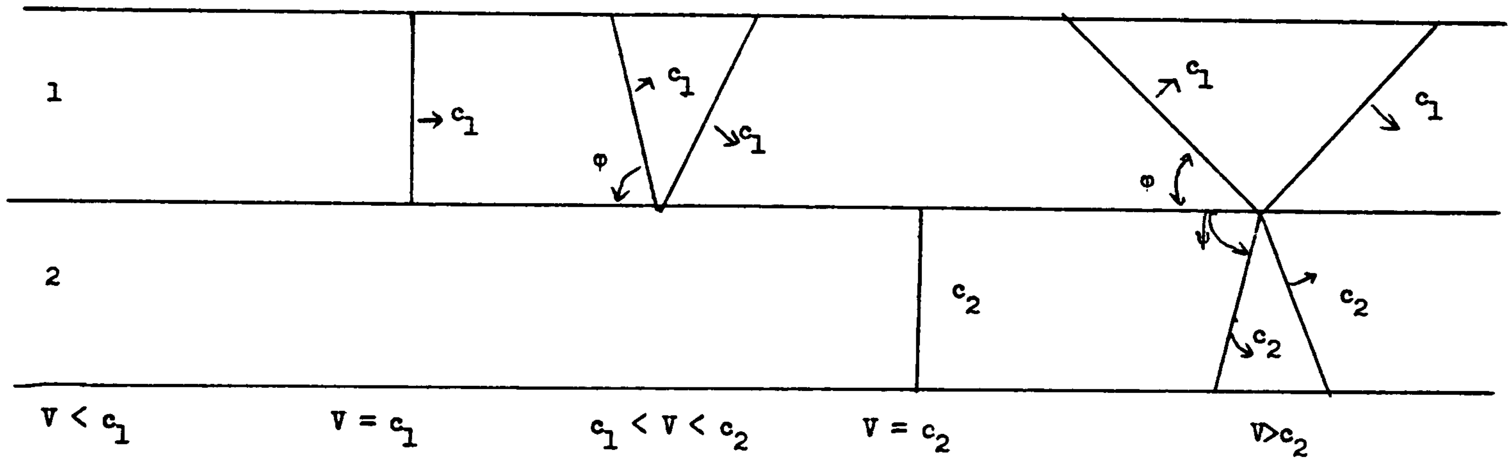

FIG. V.1. Schematic plot of the Eshelby-Stroh discontinuities for a moving screw dislocation at different dislocation velocities. The velocity c_1 is assumed to be less than c_2.

($\tan \varphi = |\beta_1|^{-1}$ and $\tan \psi = |\beta_2|^{-1}$).

angle with the surface of the order of $\tan^{-1} |1/\beta_i|$. When a unit length of dislocation moves a unit distance, an amount of energy equal to $\tau_o b|\beta_i|$ is dissipated and thus the E-S discontinuity produces a retarding stress σ_r equal to

$$\sigma_r \sim \tau_o |\beta_i| \tag{V.12}$$

Since τ_o is the theoretical shear stress, it seems to be impossible to get to stresses much above $\sqrt{2}\, c_i$. At greater velocities the retarding stress is greater than the stress which the slip plane can support.

Because of the retarding stress σ_r, it may not be possible to bring a dislocation from zero velocity gradually up to and beyond the velocity c_2, if the velocity c_1 is much smaller than c_2. Equation (V.6) shows, however, that if the dislocatiion is set running at the velocity c_2, the displacements above the slip plane can be set equal to zero. An E-S discontinuity thus exists only below the slip plane, as shown in Fig.V.1. Since the normal of this discontinuity is in the direction of dislocation motion, no energy dissipation is involved and the dislocation can run without energy dissipation. (Other energy dissipation mechanisms, such as that due to dispersion (Eshelby, 1956,) will still operate.) The velocity c_2 thus is singular in that the dislocation again acts like a subsonic dislocation. This type of singular dislocation velocity in the supersonic region was first noticed by Eshelby (1949) for edge dislocation. This type will be considered again in the next section on edge dislocations. Eshelby's singular

dislocation velocity probably approximates the velocity at which diffusionless transformations take place. It represents a fast dislocation velocity at which energy dissipative processes from the E-S discontinuities are minimized. (Of course other dissipative processes still can occur.)

Above the velocity c_2 the E-S discontinuities reappear above the slip plane, as shown in Fig. V.1.

EDGE DISLOCATION

The problem of an edge dislocation moving on an interface separating two different elastic media is more complicated but more interesting than that of the moving screw dislocation. The constants appearing in Eqns. (V.4) for the elastic displacements are simply evaluated. If Eqns. (V.4) are placed into Eqns. (V.2) one finds that

$$C_i = \gamma_i A_i \tag{V.13a}$$

$$D_i = \beta_i^{-1} B_i \tag{V.13b}$$

The condition that the displacements describe an edge dislocation gives the equation

$$A_1 + B_1 + A_2 + B_2 = 2 \tag{V.14a}$$

The condition that no line forces act at the dislocation core results in

$$\mu_1(A_1 \alpha_1^{\ 2} + B_1) + \mu_2(A_2 \alpha_2^{\ 2} + B_2) = 0 \tag{V.14b}$$

where $\alpha_i^{\ 2} = 1 - V^2/2c_i^{\ 2}$

The requirement that the stresses be continuous across the slip plane produces the equation

$$\mu_1(\gamma_1 A_1 + \alpha_1^2\beta_1^{-1}B_1) - \mu_2(\gamma_2 A_2 + \alpha_2^2\beta_2^{-1}B_2) = 0 \quad \text{(V.14c)}$$

Finally, the condition that the elastic displacements be continuous across the slip plane gives

$$\gamma_1 A_1 + \beta_1^{-1}B_1 - \gamma_2 A_2 - \beta_2^{-1}B_2 = 0 \qquad \text{(V.14d)}$$

When Eqns. (V.14) are solved for A_1 etc., one obtains

$$A_1 = \frac{2}{\Delta\beta_1\beta_2}\left\{\alpha_1^2\mu_2(\mu_1\alpha_1^2 - \mu_2\alpha_2^2) + \gamma_2\mu_2\,(\mu_1\beta_1 + \mu_2\beta_2 - \mu_1\beta_1\alpha_1^2 - \mu_1\beta_1\gamma_2^2)\right\} \qquad \text{(V.15a)}$$

$$B_1 = \frac{2}{\Delta\beta_2}\left\{\gamma_1\mu_2\alpha_2^2(\mu_2\alpha_2^2 - \mu_1) + \gamma_1\gamma_2\mu_2\beta_2(\mu_1 - \mu_2) + \gamma_2\mu_2\mu_1\alpha_1^2(\alpha_2^2 - 1)\right\} \qquad \text{(V.15b)}$$

where

$$\Delta = \frac{1}{\beta_1\beta_2}\Big\{-(\mu_2\alpha_2^2 - \mu_1\alpha_1^2)^2 - \mu_1\mu_2(\gamma_1 + \gamma_2)(\beta_1\alpha_2^2 + \beta_2\alpha_1^2)$$

$$+ (\mu_1\gamma_1 + \mu_2\gamma_2)(\mu_1\beta_1 + \mu_2\beta_2) - \gamma_1\gamma_2\beta_1\beta_2(\mu_1 - \mu_2)^2$$

$$- \mu_1\mu_2(\beta_1 + \beta_2)(\gamma_1\alpha_2^2 + \gamma_2\alpha_1^2) + \alpha_1^2\alpha_2^2\mu_1\mu_2(\beta_1\gamma_2 + \beta_2\gamma_1)$$

$$+ \beta_1\gamma_1\mu_2^2\alpha_2^4 + \beta_2\gamma_2\mu_1^2\alpha_1^4\Big\}$$

To obtain A_2 and B_2 one merely interchanges the subscripts 1 and 2 in these equations.

STRESS ON THE DISLOCATION SLIP PLANE AND THRESHOLD DISLOCATIONS VELOCITY

With these values of the constants A_1, etc., one obtains the

following expression for the shear stress σ_{xy} acting on the slip plane.

$$\sigma_{xy} = \frac{2b\mu_1\mu_2}{\pi x\ \beta_1\beta_2\Delta}\left\{\gamma_2(1-\alpha_2^{\,2})\mu_1(\gamma_1\beta_1 - \alpha_1^{\,4}) + \gamma_1(1-\alpha_1^{\,2})\mu_2(\gamma_2\beta_2 - \alpha_2^{\,4})\right\} \qquad (V.16)$$

It has been found from a study of edge dislocations moving in an isotropic material (Weertman, 1961) that as the dislocation velocity is increased the shear stress on the slip plane decreases until it becomes zero at the Rayleigh wave velocity. At greater velocities it increases with increasing velocity but has a negative value. For the type of dislocation under considerations the velocity at which the shear stress on the slip plane goes to zero can be found by setting Eqn. (V.16) equal to zero. The following equation is obtained:

$$\frac{\mu_1^{\,2}}{\gamma_1\rho_1}(\gamma_1\beta_1 - \alpha_1^{\,4}) + \frac{\mu_2^{\,2}}{\gamma_2\rho_2}(\gamma_3\beta_2 - \alpha_2^{\,4}) = 0 \qquad (V.17)$$

The velocity which satisfies that equation is the threshold velocity (Teutonico's term (1962b)) separating the region of normal dislocation behavior (in which dislocations of like sign on the same slip plane repel one another) from the region of anomalous behavior (dislocations of like sign attract one another).

The equation $\quad \gamma_1\beta_1 - a_1^{\,4} = 0 \qquad$ (V.18)

determines the Rayleigh surface wave velocity in the isotropic medium above or below the slip plane. Hence one can see from

Eqn. (V.17) that the threshold velocity lies between the Rayleigh wave velocities of each medium. The threshold velocity defined by Eqn. (V.17) is <u>not</u> equal to the Stoneley wave velocity*, contrary to what one might have assumed from the fact that the threshold velocity is equal to the Rayleigh wave velocity when the elastic properties and densities of the two media are identical.

It is not always possible to find a velocity which satisfies Eqn. (V.17). Figure V.2 illustrates this situation for the simple case in which the longitudinal sound velocity in each medium is very much larger than both c_1 and c_2 so that both γ_1 and γ_2 are equal to 1 for velocities near c_1 and c_2. The densities in the two media are assumed to be equal. One can see from the figure that the occurrence of a threshold velocity is limited to a narrow range in the variables μ_1 and μ_2 ($(\mu_2/\mu_1)^{1/2}$ varying from 1 to 1.193). If μ_2/μ_1 is outside this range the anomalous velocity region cannot exist.

SUPERSONIC EDGE DISLOCATION

Above the slower of the two transverse sound velocities the edge dislocation is in a supersonic region. As in the case of a screw dislocation Eshelby-Stroh discontinuities will arise along the planes $x' = \pm|\gamma_1|y$ when γ_1 also is an imaginary quantity. The behavior to be expected is shown in Fig. V.3 for the case in which $c_{\lambda 2} > c_{\lambda 1} > c_2 c_1$. There are no discontinuities below the

*Stonely waves (Ewing, Jardetzky, and Press, 1957) are the surface waves which propagate along an interface separating two semi-infinite media of differing elastic properites. They are simply a generalization of the Rayleigh wave which propagates along a free surface.

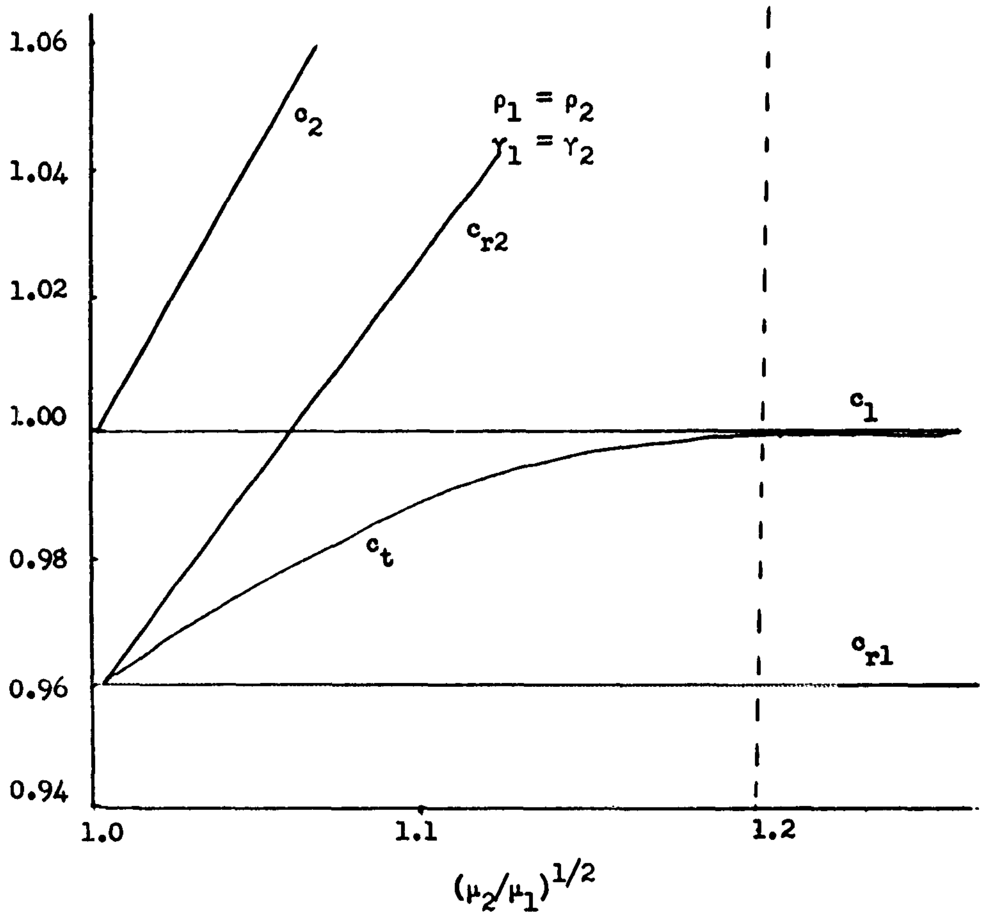

FIG. V.2 Plot of the threshold velocity c_t versus $(\mu_2/\mu_1)^{1/2}$ The velocity is expressed in units of c_1. Also shown is the variation of the transverse sound velocities c_1 and c_2 and the two Rayleigh surface wave velocities c_{r1} and c_{r2}.

velocity c_1 except at the dislocation core. At the velocity c_1 and E-S discontinuity occurs whose normal is the direction of dislocation motion. In contrast to the case of the screw dislocation, the stresses about an edge dislocation in the upper plane are (when linear elasticity is applied) infinite everywhere rather than in a region limited to the plane $x' = 0$. This infinity in energy is thus of a different kind from that of a screw dislocation. Stroh has pointed out that the difference is due to a resonance phenomenon. At velocities between c_1 and c_2 the normals of E-S discontinuities make an angle with the direction of dislocation motion. The dislocation experiences a retarding force given again by Eqn. (V.12). At higher velocities other E-S discontinuities appear, as shown in the figure.

ESHELBY'S SINGULAR DISLOCATION VELOCITY

Eshelby (1949) showed that an edge dislocation in an isotropic medium can move at a velocity $\sqrt{2}c$ without any E-S discontinuity appearing in its displacement field. This velocity is in a supersonic range and yet the dislocation will not radiate energy. The reason why the dislocation can exhibit behavior can be seen from Eqns. (V.14) for the case of $\mu_1 = \mu_2, \lambda_1 = \lambda_2$, and $\rho_1 = \rho_2$. In this situation Eqns. (V.14c) and (V.14d) are identically zero since $A_1 = A_2$ and $B_1 = B_2$. Now B_1 and B_2 of Eqns. (V.14a) and (V.14b) could be set equal to zero (and thus eliminate the terms of Eqn. (V.4) which contain the E-S discontinuity when $c < V < c_\lambda$) and still these equations may be satisfied provided the velocity is such that $a_1 = a_2 = 0$. This velocity is $\sqrt{2}c$. At this velocity

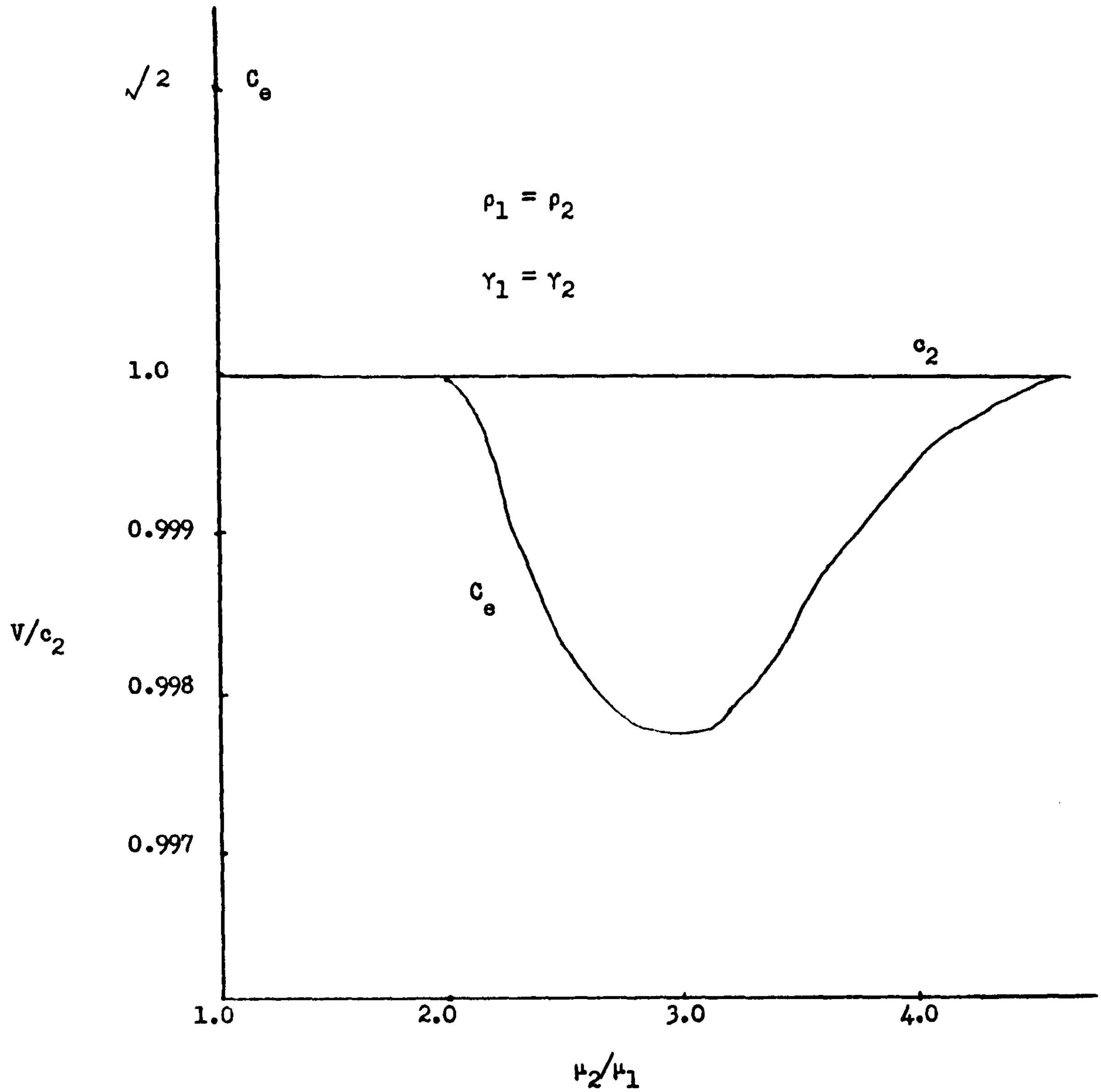

FIG. V.3. Schematic plot of the Eshelby-Stroh discontinuities for a moving edge dislocation at different dislocation velocities. It is assumed that $c_1 < c_2 < c_{\lambda 1} < c_{\lambda 2}$.

no discontinuities appear in the stress and displacement field and all of Eqn. (V.14) are satisfied.

Consider now what happens to Eshelby's singular dislocation velocity in the edge dislocation being discussed. For this singular velocity to exist, it must be possible to set any of the A_i and B_i of Eqns. (V.14) equal to zero when the terms multiplied by A_i and B_i of Eqns. (V.4) contain E-S discontinuities. If the dislocation velocity lies between c_1 and c_2, and B_1, therefore, is set equal to zero, Eqns. (V. 4) cannot, in general, hold for any value of V since we have four equations in three unknown quantities. However, it is possible for all equations to hold if the determinate of the coefficients of Eqns. (V.14b) through (V.14d) is zero. Thus the following equation for Eshelby's singular dislocation velocity is:

$$\frac{\mu_2}{\mu_1} (\gamma_2\beta_2 - \alpha_2{}^4) \; - (\gamma_2\beta_2 - \alpha_2{}^2) \; \frac{\gamma_2}{\gamma_1} \alpha_1{}^2(1 - \alpha_2{}^2) = 0 \qquad \text{(V.19)}$$

For the case in which $\rho_1 = \rho_2$ and $c_{\lambda i}$ is very much larger than both c_1 and c_2 so that as $\gamma_1 = \gamma_2 = 1$ for the velocities of interest, it is found that as μ_2/μ_1 is varied from 1 to infinity the Eshelby singular velocity c_e varies as shown in Fig. V.4. The singular velocity can exist when $\mu_1 = \mu_2$. When μ_2 is slightly larger than μ_1 no singular velocity occurs. In the range $2 \leq \mu_2/2_1 \leq 4$ a singular velocity does exist. Its value is almost equal to the velocity c_2. In contrast with the case of the Eshelby singularity in the screw dislocation, this singular velocity

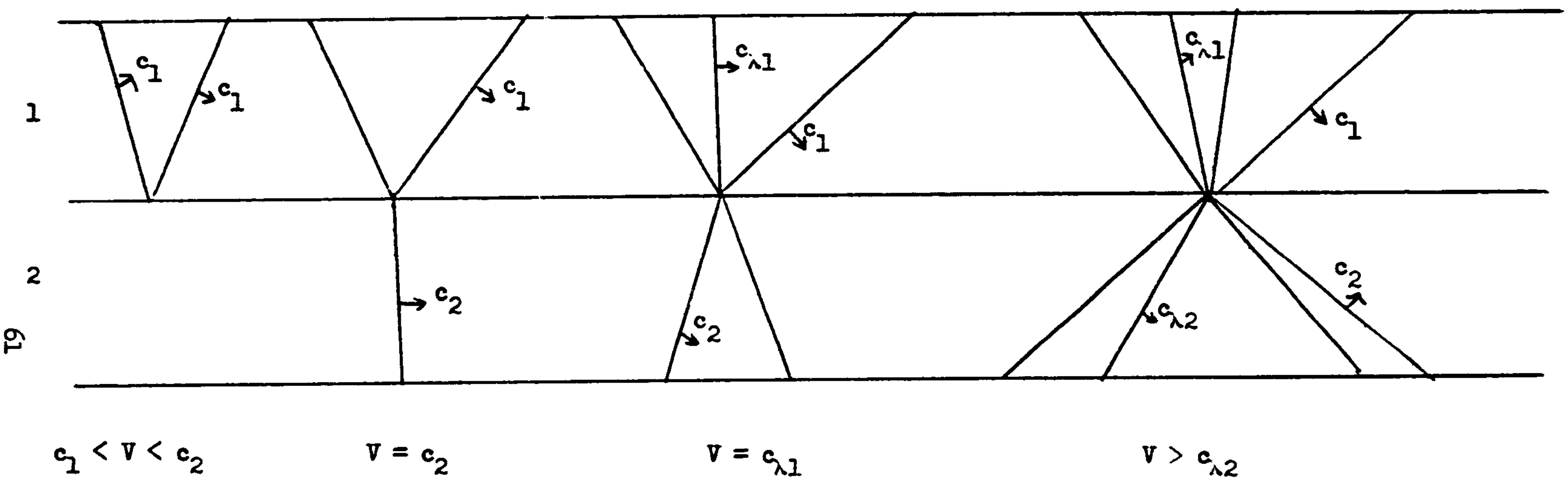

FIG. V.4 Variation of Eshelby's singular velocity c_e as a function of μ_2/μ_1. It is assumed that $\gamma_2 = \gamma_1$ and $\rho_2 = \rho_1$. The velocity is expressed in units of c_2.

has no infinite stresses associated with it.

If $\mu_1 = \mu_2$ it is not possible to have a singular velocity which is larger than both c_1 and c_2. However, if $\mu_1 = \mu_2$ (but ρ_1 is not necessarily equal to ρ_2) it is possible to have a singular velocity greater than c_1 and c_2. In Eqns. (V.14) B_1 and B_2 can be set equal to zero. When this is done, one obtains

$$\gamma_1 A_1 = \gamma_2 A_2 \qquad (V.20)$$

For Eqn. (V.14b) to hold the velocity must be such that

$$\alpha_1^2 + (\gamma_1/\gamma_2)\alpha_2^2 = 0 \qquad (V.21)$$

This equation defines the singular velocity. When $\gamma_1 = \gamma_2 = 1$ this velocity is given by

$$c_e^2 = \frac{4c_1^2 c_2^2}{c_1^2 + c_2^2} = \frac{4\mu}{\rho_1 + \rho_2} \qquad (V.22)$$

SUMMARY

The behavior of dislocations moving on the interface between two different elastic media generally is found to be what one would expect from the studies of dislocations moving in isotropic material. The slowest sound velocity sets an upper limit (apart from the Eshelby singular velocity) to the speed of the dislocation since, according to linear elasticity theory, the energy becomes infinite at this point. If it is assumd that a crystalline material can support only a finite stress, it can be shown that supersonic solutions exist. Eshelby and Stroh have pointed out this fact.

Because of damping caused by the generation of sound waves, the supersonic dislocation velocities probably are limited to velocities not much beyond $\sqrt{2}$ times a sound velocity.

The Eshelby singular dislocation velocity may or may not exist. Its existence depends on the values of the elastic constants and the density in each medium. Because energy dissipation processes are minimized at the Eshelby singular velocity, this velocity could be that at which transformation dislocations run on diffusionless transformation interfaces. It would be interesting, therefore, to see if fast diffusionless transformations take place in materials whose elastic constants do not permit the existence of an Eshelby singular velocity.

The threshold velocity separating a normal from an anomalous velocity region also may or may not exist. It will exist only if the two transverse sound velocities in the two isotropic media above and below the slip plane lie close to each other.

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